EDITOR ANALYST:

What you represent as my correction [see p. 176] to 352 is not accepted. The only correction I can admit to my original solution is that of introducing the probability of selecting a second chord equal to the first, which is $2d\varphi \div \pi$, as given in your first letter of objeetion to that solution. You gave no reason that $2d\varphi \div \pi$ is this probability. The reason however is given at length in my revised solution. And the reason for the contingent probability $2\varphi \div \pi$, is given at length in my original solution.

My objection to your construction on page 160 is that the chords should be drawn from every point on the circumference of the circle AC to every other point on that circle at the distance $2AC \sin \varphi$ from the first one, instead of tangent at every point of circumference of the circle CD. The whole number in the one case being proportional to $2\pi AC$, and in the other $2\pi CD$.

R. J. ADCOCK.

Roseville, Ill., Sept. 24, 1881.

INFINITE SERIES.

BY PROF. L. G. BARBOUR, RICHMOND, KY.

To find the sum of an infinit number of terms of the series

$$\frac{1}{1.3}+\frac{1}{5.7}+\frac{1}{9.11}+\frac{1}{13.15}+\frac{1}{17.19} \text{ \&c.}$$

First calculate directly the sum of these five terms $= .380229950$. Next to find the sum of $\frac{1}{21.23}$ &c. Let

$$S=\frac{1}{21.23}+\frac{1}{25.27}+\frac{1}{29.31} \text{ \&c.}$$

By the formula

$$S=\frac{1}{p}\Big(\frac{q}{n}-\frac{q}{n+p}\Big),$$

we have

$$S=\tfrac{1}{2}\Big\{\frac{1}{21}-\frac{1}{23}+\frac{1}{25}-\frac{1}{27}+\frac{1}{29}-\frac{1}{31} \text{ \&c.}\Big\}.$$

Now since $\frac{1}{23}-\frac{1}{25}$ is *nearly* midway between $\frac{1}{21}-\frac{1}{23}$ and $\frac{1}{25}-\frac{1}{27}$, assume $\frac{1}{23}-\frac{1}{25}=\frac{1}{2}(\frac{1}{21}-\frac{1}{23}+\frac{1}{25}-\frac{1}{27})$. So also assume $\frac{1}{27}-\frac{1}{29}=\frac{1}{2}(\frac{1}{25}-\frac{1}{27}+\frac{1}{29}-\frac{1}{31})$, and so on.

$$\therefore\ 2S=\frac{1}{21}-\frac{1}{13}+\frac{1}{25}-\frac{1}{27}+\frac{1}{29}-\frac{1}{31}.$$

Then the sum of the new series $\frac{1}{23}-\frac{1}{25}+\frac{1}{27}-\frac{1}{29}$ &c. $=\frac{1}{2}(\frac{1}{21}-\frac{1}{23})+\frac{1}{25}-\frac{1}{27}+\frac{1}{29}-\frac{1}{31}$ &c. $=2S-\frac{1}{2}(\frac{1}{21}-\frac{1}{23})$. Adding the two series we have

$4S-\frac{1}{2}(\frac{1}{21}-\frac{1}{23})=(\frac{1}{21}-\frac{1}{23}+\frac{1}{23}-\frac{1}{25}+\frac{1}{25}$ &c.); $\therefore\ 4S=\frac{1}{2}(\frac{1}{21}-\frac{1}{23})+\frac{1}{21}$ $=\frac{3}{2}\cdot\frac{1}{21}-\frac{1}{2}\cdot\frac{1}{23}$. $S=\frac{3}{8}\cdot\frac{1}{21}-\frac{1}{8}\cdot\frac{1}{23}=.012422360$

Add sum of first five terms	= .380229950
	.392652310
True result	= .392699082
Error	.000047372

A closer approximation could be obtained by going farther down the line and adding directly the first six, seven or more terms. Thus take six terms, the last one being $\frac{1}{21.23}$; add $\frac{3}{8}$ of $\frac{1}{25}$ and subtract $\frac{1}{8}$ of $\frac{1}{27}$ and we get the sum = .392670714; the error = .000028368.

But there is a more rapid method of approximation. The sum of the first six terms is .382300343. Then $\frac{1}{25\cdot27}+\frac{1}{29\cdot31}$ &c. $=\frac{1}{2}(\frac{1}{25}-\frac{1}{27}+\frac{1}{29}-\frac{1}{31}$ &c.) $=\frac{1}{2}\cdot\frac{1}{25}-\frac{1}{2}(\frac{1}{27}-\frac{1}{29}+\frac{1}{31}$ &c.). Add $\frac{1}{2}\cdot\frac{1}{25}$ to the sum of the first six terms and we get .402300344. We have now to subtract $\frac{1}{2}(\frac{1}{27}-\frac{1}{29}+\frac{1}{31}-\frac{1}{33}$ &c) $=\frac{3}{8}\cdot\frac{1}{27}-\frac{1}{8}\cdot\frac{1}{29}$.

.402300343	
—.013888888	
.388411455	
+.004310345	
.392721800,	a result too large by .000022718
.392670714	= former result
.392696257	= average
.392699082	= true result
.000002825	= error.

The reader may be curious to know how the true result referred to is obtained. Develope $\tan^{-1}x$ by Maclaurin's theorem. It is equal to $x-\frac{1}{3}x^2+\frac{1}{5}x^3-$ &c. Let the arc $=45^\circ=\frac{1}{4}\pi$; $\therefore x=1$;

$$\therefore \text{arc } 45^\circ = 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7} \text{ \&c.} = 2\left\{\frac{1}{1.3}+\frac{1}{5.7}+\frac{1}{9.11} \text{ \&c.}\right\}.$$

Hence divide 3.1415962535 by 8 and we find

$$\frac{1}{1.3}+\frac{1}{5.7}+\frac{1}{9.11} \text{ \&c.} = .392699082 = \text{arc of } 22\tfrac{1}{2}^\circ.$$

But this method by the Calculus is not general. The Algebraic method just given is not beyond the reach of the student in Algebra, and admits moreover of quite a wide extension.

Thus to find the sum of

$$\frac{1}{1.3}+\frac{1}{7.9}+\frac{1}{13.15}+\frac{1}{19.21} \text{ \&c.}$$

The sum of the first four terms = .356840768, and by a process analogous to the foregoing we find $S=$ .007160494,

and the required sum is .364001262.